

AD-783 442

**THE MECHANICAL BEHAVIOR OF PLASTICS AND  
METALS AT HIGH RATES OF LOADING**

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**Prepared for:**

**Army Research Office**

**19 July 1974**

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Unclassified

Security Classification

AD - 783442

## DOCUMENT CONTROL DATA - R &amp; D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified.)

1. ORIGINATING ACTIVITY (Corporate author)		2a. REPORT SECURITY CLASSIFICATION	
Brown University		Unclassified	
		2b. GROUP	
		NA	
3. REPORT TITLE			
The Mechanical Behavior of Plastics and Metals at High Rates of Loading			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)			
Final report			
5. AUTHOR(S) (First name, middle initial, last name)			
H. Kolsky			
6. REPORT DATE		7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
July 19, 1974		34	
8a. CONTRACT OR GRANT NO.		8b. ORIGINATOR'S REPORT NUMBER(S)	
DA-31-124-ARO-D-358			
9. PROJECT NO.		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
DA-ARO-D-31-124-71-G59			
10. DISTRIBUTION STATEMENT			
Approved for public release; distribution unlimited.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY	
		U. S. Army Research Office Box CM, Duke Station Durham, North Carolina 27706	
13. ABSTRACT			
This is the final report which summarizes the work carried out at Brown University under the above contract.			

NATIONAL TECHNICAL  
INFORMATION CENTER

11-11-73

DD FORM 173

REPLACES DD FORM 173, 1 JAN 65, WHICH IS  
OBSOLETE FOR ARMY USE.

Security Classification

**KEY WORDS:**

Mechanical properties  
Plastics  
Metals  
Dynamic loading  
Polymers  
Spheres  
Impact tests  
Shock waves  
Elastomers  
Wave propagation  
Stresses  
Zinc  
Fiber composites

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AUTHORIZED DOCUMENTS.

## Introduction

The research program carried out here at Brown with the support of AROD has covered a wide range of interrelated problems, all of which are concerned with the dynamic mechanical response of non-metallic solids (especially plastics) as well as metals under conditions of rapid loading.

A list of the titles of the reports issued under this grant together with their relevant abstracts is appended herewith. A list of the papers which have appeared in the open literature is also given together with the thesis titles of successful Ph.D. and M.S. candidates who have received support from this grant. These lists and abstracts give a summary of the details of the separate research projects which have been undertaken and it may be seen that these fall into a number of headings, a list of which is given below:

- 1) Impact of hard spheres on blocks and plates of polymers and of metals.
- 2) Tensile shock waves in rubber.
- 3) Viscoelastic 'cross over effect'.
- 4) Stress wave propagation in geological materials.
- 5) Stress wave propagation in zinc single crystals.
- 6) Torsional oscillations of pre-stressed polymer and rubber specimens.
- 7) Torsional split Hopkinson bar apparatus.
- 8) Wave propagation in fiber reinforced composites.
- 9) Flexural waves set up at boundaries between two rods.

Brief summaries of significant findings in each of these fields is given below.

Summaries of significant findings in the different fields of study

1) Hertzian impacts

An extremely simple experimental technique for studying the response of solid specimens to large forces applied for durations of only a few tens of microseconds is to arrange for steel balls to impinge on the flat surfaces of blocks of the material under study and observe the waves sent out and the energy dissipated during the impact. The simplest theoretical approach is to consider the deformation to be similar to that found quasi-statically and this gives a reasonably close approximation for large elastic blocks. When however the specimen is dissipative or is in the form of a plate, a more detailed analysis is called for. The results of such experimental and theoretical studies with polymeric specimens are described in reports 1,2,3 (which have since been published) while a comparison between the observations and the results predicted by linear viscoelastic theory are given in report no. 7 which has also now appeared in the open literature. The corresponding problem for metallic specimens is described in report no. 5 which has since been published in the International Journal of Solids and Structures.

2) Shock Waves in Rubber

A novel phenomenon has been discovered in the propagation of incremental tensile pulses in stretched rubber specimens. It has been shown that under these conditions tensile shock fronts are

developed as a result of the nonlinear mechanical response of the material. Similarly when a compressive pulse is propagated along a stretched rubber specimen a shock tail is developed. These results are described in reports 14 and 19 which have been published in the open literature.

### 3) Viscoelastic 'cross over' effect.

When a mechanical pulse reaches the boundary between two collinear elastic rods two pulses are generated, one is reflected back along the rod along which the incident pulse was travelling and a second 'transmitted' pulse is propagated along the second rod. It is found under these conditions that all three pulses have the same profiles but the reflected pulse will have respectively the same sign or the opposite sign (i.e. compressive or tensile) depending on whether the acoustic impedance  $Z_2$  of the second rod is higher or lower than that of the incident rod  $Z_1$ . Now if the second rod is viscoelastic we may have  $Z_1 > Z_2$  at low frequencies and  $Z_1 < Z_2$  at high frequencies. Under these conditions a unidirectional pulse is reflected as a sigmoidal one, this theoretically predicted behavior was named the 'viscoelastic cross-over effect' and has been demonstrated experimentally with rods of Polyvinyl Chloride. The results are described in report no. 18 and have been published in the Journal of the Acoustical Society.

Analogous effects have been studied for the measurement of mechanical properties by observing the reflected pulses between an elastic rod and a specimen either in the form of a rod collinear with the first rod or in a block. With block specimens it was

shown that 'cross-over' effects can be observed even when the material of the block obeys Hooke's Law. The results of these investigations are described in reports nos. 10, 12, and 20, the last report was the basis of a recent publication in the Journal of the Acoustical Society

4) Stress wave propagation in clay.

Many geological materials behave in a nonlinear time dependent manner and some experiments have been carried out on the propagation of longitudinal stress pulses along rods of clay. The results of this work is described in report no. 6 which was published in Experimental Mechanics.

5) Dynamic deformation of zinc single crystals.

The deformation of zinc single crystals by longitudinal stress pulses was studied, and it was shown that the stress-strain curve is concave upwards and that yielding begins at about 2500 psi. The work is described in report no. 11 and an account was published in 'Nature'.

6) Torsional oscillations of prestressed polymer and rubber specimens.

The effect of large longitudinal prestrain and torsional deformations has provided an interesting field of study. This work is described in reports 8 and 16 which dealt with the behavior of polyethylene and polyvinyl chloride respectively and is being continued at present on specimens of natural rubber where the effects of crystallation show a marked effect.

7) Torsional split Hopkinson-Bar apparatus.

The split Hopkinson Bar invented by the Chief Investigator of this contract about twenty years ago, has become one of the principal experimental techniques for the study of rapid longitudinal deformation. The extension of this technique to shear deformations by the use of torsional instead of longitudinal waves is described in report no. 9 which was the basis of part of the doctoral thesis of James Phillips.

8) Wave propagation in fiber-reinforced composites.

The mechanical behavior of fiber-reinforced composites is closely analogous to that in crystalline materials and the propagation of stress waves in them shows a number of interesting features some of which have not been observed in normal crystalline solids. A study of the behavior of these materials is given in report no. 21 where the effects of geometrical dispersion of waves in rods of these materials are described.

9) Flexural waves generated at the boundaries between elastic rods.

When a longitudinal elastic wave is reflected at the boundary between two non-collinear rods four waves are, in general, generated, a longitudinal and a flexural wave are reflected back along the first rod and a wave of each type is transmitted to the second. A theoretical study of the effects together with a comparison with experimental observations is given in reports 15 and 17. The results described in the first of these were described in a paper published in the Journal of Applied Mechanics while the results described in the second were published in a



paper in the International Journal of Solids and Structures. More recent work on this problem has just been published in the Journal of the Acoustical Society of America. The response of solids to such steep fronted flexural waves has shown interesting new phenomena in the fracture of brittle solids.

Publications on work supported by this Grant

- H.H. Calvit, Experiments on rebound of steel balls from blocks of polymer, *Journal of Mechanics and Physics of Solids* 15, 141-150 (1967).
- H.H. Calvit, Numerical solution of the problem of impact of a rigid sphere onto a linear viscoelastic half-space and comparison with experiment, *Int. J. Solids Structures*, Vol. 3, 951-966 (1967).
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- L.R. Oliver, R.W. Armstrong, R.J. Clifton and H. Kolsky, Cleavage of zinc crystals induced by stress waves, *Nature* 216, 910 (1967).
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D.Y. Hsieh and J.P. Lee, Experimental study of pulse propagation in curved elastic rods, J. Acoust. Soc. of America, Vol. 54 No. 4, 1052 (1973).

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Dennis Rader (1966) A Stress Wave Study of Brittle Fracture.

Yu-Min Tsai (1967) A Theoretical and Experimental Study of Hertzian Fractures in Glass.

James Phillips (1969) Part I: Stress Pulses Produced by Brittle Fracture of Rods.

Part II: A Method for Determining Material Properties at High Rates of Shearing Strain.

Amnon Meer (1969) Longitudinal Wave Propagation in Rods with Interfaces and Rods of Non-Uniform Cross-Section.

J. P. Walsh (1969) An Experimental Investigation of the Dynamic Tangent Moduli of Polyethylene.

James P. Lee (1971) The Production of Flexural Waves in Bent Rods, and the Fractures Produced by Them.

Master's theses of students supported by this Grant.

Sankar Chandra Das (1965) Measurements of the dynamic shear modulus and internal friction of pre-stressed and pre-strained polyethylene.

James Phillips (1967) Impact of rigid sphere on a viscoelastic plate.

L. R. Oliver (1967) Stress wave produced cleavage of zinc monocrystals with optical and X-ray investigations of associated deformation.

Edmund J. Sullivan, Jr. (1969) High strain-rate compression of zinc single crystals.

Bernard J. Reilly, Jr. (1971) Dynamic tangent moduli of polyvinyl chloride subjected to large strains.

S. Boucher (1972) Reflection of pulses at the interface between an elastic rod and an elastic half-space.

Peter J. Arsenaux, Jr. (1973) An experimental investigation of stress waves in rods of a fiber-reinforced composite.

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## Tech. Report No. 1

Experiments on rebound of steel balls from blocks of polymers

H.H. Calvit

Abstract. Experiments are performed in which steel balls are impacted onto blocks of polymer. The time of contact and height of rebound are recorded as functions of ball size, velocity of impact and temperature of the specimen. These results are compared with results of similar tests as well as those of different methods of dynamic testing.

## Tech. Report No. 2

Numerical solution of the problem of impact of a rigid sphere onto a linear viscoelastic half-space and comparison with experiment.

H. H. Calvit

Abstract. The pair of coupled nonlinear integro-differential equations which describe the impact of a rigid sphere onto a viscoelastic half-space are solved numerically. Using data obtained from free torsional vibration experiments, the rebound of a steel ball from a block of polymer is predicted for various temperatures. These results are compared with rebound experiments performed on the same material. The viscoelastic solution is also compared with elastic solutions and an approximate solution. Some conclusions are drawn on using the rebound method as a means of dynamic testing.

Tech. Report No. 3

The Impact Problem for a Rigid Sphere and  
a Viscoelastic Plate

J. W. Phillips and H. H. Calvit

Abstract.-- The impact problem involving a rigid sphere and a large, thin, linear viscoelastic plate is treated theoretically and experimentally; it is found that while a precise separation of the two anelastic effects involved is possible when the plate material properties are known a priori, the reverse procedure of using experimentally observed times of contact and coefficients of restitution in order to predict plate material properties is not easily accomplished.

Tech. Report No. 4

The Numerical Solution of a Problem in the  
Propagation of Plastic Waves of Combined Stress

D. L. Vitiello and R. J. Clifton

Abstract .-- In / <sup>earlier work</sup> the second author considered the problem of combined longitudinal and torsional plastic wave propagation in a thin-walled tube of a strain rate independent material. The governing equations were written as a system of quasi-linear, symmetric, hyperbolic, partial differential equations of first order for which it was suggested that solutions, satisfying arbitrary initial and boundary conditions, could be obtained by using difference equations along characteristics. In the present paper such a difference scheme is developed based on the difference method originated by Courant, Isaacson and Rees.

The accuracy of the difference method is studied by comparing the difference solution with the exact solution for a problem for which the latter solution is known. As an application of the difference method the solution is obtained for the case of a tube of finite length, fixed at one end, and subjected to step loading in normal stress and shear stress at the opposite end. Stress and velocity profiles are presented as functions of both distance and time; regions of unloading are located.

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Tech. Report No. 5

Stress Waves Produced by Impact on the  
Surface of a Plastic Medium

by

Y. M. Tsai

Abstract. Experiments were carried out to observe the effect that local plastic yielding has on the radial surface strain waves produced by the impact of a hard sphere on the surface of a steel block. At impact velocities slightly greater than that required first to initiate local plastic yielding, the effect was almost imperceptible close to the area of impact, but became observable at distant points. At higher velocities of impact, however, the experimental results clearly show the presence of plastic yielding as a sudden change in the slope of the strain waves for all distances of travel.

In the development of the theory the surface displacements of an elastic half-space were written as integrals over the first derivative of an arbitrary vertical loading  $f(t)$  applied at a point on the free surface. It is shown that at large distances  $r$  away from the point of impact the amplitude of the radial surface strain generated by  $f(t)$  is inversely proportional to  $r^{1/2}$ . However, the radial surface strains excited by finite jumps of  $f(0)$  and  $f'(0)$  are shown to decay as  $r^{-2}$  and  $r^{-1}$  respectively for large values of  $r$ . On the basis of the theory developed here, the slope of the applied forcing function is shown to vary rapidly when local plastic yielding has occurred.



Tech. Report No. 6

Some Wave Propagation Experiments in Plasteline Clay Rods

by

H. H. Calvit, D. Rader and J. Melville

Abstract. Compressional stress pulses have been propagated in plasteline clay rods by detonating small charges of lead azide on one end. A capacitance gauge at the other end has been used to measure particle displacement associated with the pulses and hence the particle velocity was obtained by differentiation. The stress shapes and amplitudes for the pulses were determined in separate experiments using composite clay-steel rods where the steel acted, in effect, as a pressure transducer. The techniques employed permitted comparatively accurate determination of some aspects of the dynamic behavior of clay. On the basis of preliminary results the behavior of clay has been compared to that of a linear viscoelastic solid with the tentative goal of studying the validity of a viscoelastic constitutive model.

Surface Wave Propagation for  
Linear Viscoelastic Solids

by

Y. M. Tsai and H. Kolsky

Abstract. Experiments have been carried out to observe the surface waves produced on large blocks of polyethylene and polymethylmethacrylate (p.m.m.) when specimens of these materials have been subjected to surface impacts of steel balls or when small lead azide charges have been detonated on the free surfaces. It has been shown that if Poisson's ratio,  $\nu$ , is taken to be a real constant the shapes of surface waves can be calculated if the shape of the initial disturbance and the viscoelastic properties of the medium are known. It has further been shown that the complex nature of  $\nu$  results in only second order effects for the materials used. It has consequently been found possible to forecast the shape of surface waves produced by the impact of steel balls. The agreement with experimentally observed shapes was found to be excellent for the p.m.m. specimen. The theoretical predictions were less accurate for the polyethylene specimen, but the predictions here agreed reasonably well with the experiments. It was not found possible to predict the surface waves produced by the explosive charges chiefly because of uncertainty of the shape of the initial disturbance. The experimental results did, however, show

two distinct groups of waves, one travelling with the dilatational velocity and the other with a velocity close to that of the shear velocity. At short distances of travel these two groups overlapped.

Tech. Report No. 8

An Experimental Investigation of the Dynamic  
Tangent Moduli of Polyethylene

by James P. Walsh

Abstract.

The research presented in this report is on the study of the behavior of the dynamic tangent moduli of polyethylene under large quasi-static deformations. This involved both the determination of the complex shear modulus  $G^*$  and the complex Youngs modulus  $E^*$  as a function of longitudinal and torsional strains.

Tubular specimens were subjected to longitudinal strains of up to 150% or torsional strains of up to 30% by applying constant loads and allowing the sample to creep. At various levels of these static strains, as the sample is undergoing creep, small oscillatory strains are superimposed on the specimen through a coil and magnet system. These small dynamic strains may be imposed in either a lateral, longitudinal, or torsional mode depending on the moduli being investigated. From the decay curve of the vibrations or the resonant frequency and half-breadth of the frequency-amplitude curve, the value of the relevant storage and loss moduli are obtained.

Upon removal of the static load, the specimen will start to recover and the moduli can be determined during the recovery cycle. In addition, the variation of the different moduli under stress relaxation were also obtained for both fixed torsional and longitudinal strains.

Both the shear modulus  $G^*$  and Youngs modulus  $E^*$  were obtained in this manner, the Youngs modulus being determined from either longitudinal or lateral vibrations of the specimen, and the shear modulus by torsional vibrations.

A Method for Determining Material Properties  
at High Rates of Shearing Strain

by

J. W. Phillips

Abstract

The details of an experimental program suitable for determining the mechanical properties of certain materials at high rates of shearing strain are given. The major contribution of this work is a description of a successful method for producing sharp torsional pulses in the main loading rod of a torsional split-Hopkinson bar, by means of the simultaneous detonation of two explosive charges at the ends of long "pre-load" bars in contact with the main loading rod. The method could be used for the study of the linear viscoelastic properties of materials like high polymers, and could also be employed in the study of the non-linear behavior of materials at high rates of shearing strain.

Experimental Technique for Determining Some Dynamic  
Properties of Viscoelastic Materials

by Amnon Meer

Abstract

In this report, an experimental technique is described, by which some basic properties of viscoelastic materials such as phase velocity, loss factor and stress relaxation function could be derived. The technique is based on the propagation and reflection of longitudinal stress pulses in a system of two bars, one of which is elastic with known properties, and the other viscoelastic with properties to be determined. It is shown that the unknown properties of the viscoelastic bar may be determined from the shapes of the incident and reflected stress pulses which travel in the elastic bar. It turns out, however, that the only material property which may be derived with reasonable accuracy is the phase velocity for the basic frequency of the pulses propagating along the bars. All the other material properties proved to be extremely sensitive to small experimental errors and hence no reliable values for them could be derived.

## High Strain-Rate Compression of Zinc Single Crystals

by

Edmund J. Sullivan, Jr.

Abstract

Stress-strain data are presented for zinc single crystals compressed along the  $[0001]$  direction at strain rates of approximately  $1 \times 10^3 \text{ sec}^{-1}$  to  $6 \times 10^3 \text{ sec}^{-1}$ . The tests were run at room temperature and at the temperature of liquid nitrogen; however, very little difference in the stress-strain behavior was observed over this temperature range. The stress-strain curves show that yielding begins below 2500 psi and that the stress-strain curve is concave upward to a stress level of 40,000 psi, which is the maximum stress obtained in these tests. The primary deformation mechanisms are twinning in the  $\{10\bar{1}2\}$  twinning mode and slip on the  $\{11\bar{2}2\} \langle \bar{1}\bar{1}23 \rangle$  slip system.

Propagation of longitudinal stress waves in elastic and linearly viscoelastic bars of variable cross-section

by Amnon Meer

Abstract. The propagation of longitudinal stress waves in elastic and linearly viscoelastic bars of slowly varying cross-section, is treated by an elementary theory. Exact solutions based on this theory are presented for cones and exponentially variable cross-sections. An approximate solution which is valid in certain regions of the bar is derived for a general bar of slowly varying cross-section. According to this solution, the amplitude of the displacement wave is inversely proportional to the square root of the local cross-sectional area of the bar. (In viscoelastic bars, the amplitude is also decaying exponentially due to the internal friction in the bar.)

The effect of radial inertia on the propagation of longitudinal stress waves in elastic cones is also discussed in this report. It is found that the phase velocity of the waves is dependent on the wavelength and the local radius of the cone. This phase velocity is represented by an expression which is similar to that derived by Rayleigh for the case of a cylindrical bar of uniform cross-section.

Another problem which is treated in this report is the reflection of stress waves at the interface between two elastic or viscoelastic cones attached end to end. It is found that during reflection, changes in pulse shapes occur, both in the elastic and the viscoelastic cases.



Tech. Report No. 13

Recent Experimental Studies of the Mechanical Response  
of Inelastic Solids to Rapidly Changing Stresses

H. Kolsky  
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Abstract

The role of experiment in elucidating the dynamic mechanical behavior of inelastic solids is discussed with particular emphasis on the type of results which can be obtained from wave propagation experiments. Recent experimental results obtained by the author and his co-workers are reviewed as is work by other investigators in the field. Viscoelastic waves, plastic waves and shock waves are considered and in particular a recent observation by the author of the generation of tensile shock waves in stretched rubber is described. The fracture behavior of brittle plastics under stress-wave loading, and some recent work by Phillips on stress waves produced by tensile fractures are mentioned.

Tech. Report No. 14

The Production of Tensile Shock Waves in Stretched Natural Rubber by H. Kolsky.

No abstract available

Tech. Report No. 15

Reflection and Transmission of Elastic Stress Waves at the  
Junction of Two Semi-infinite Rods which are at an Angle to  
Each Other

J. P. Lee and H. Kolsky

Abstract

When a longitudinal stress pulse impinges on the interface of two semi-infinite rods which are joined at an angle to each other, it will, in general, produce four separate waves, namely, longitudinal and flexural waves in both rods. In this paper Fourier techniques are employed to predict these transmitted and reflected wave shapes at different angles of intersection. Timoshenko theory and one-dimensional bar theory are used to describe the lateral and longitudinal vibrations of the beams. An experimental investigation of the phenomena was carried out, and the experimental results from this were compared with the numerical theoretical predictions. It was found that for longitudinal waves the agreement was remarkably good. Except in very small regions, the predicted shapes of the flexural waves were also found to approximate very closely to the experimental observations. It was found that a critical angle of intersection exists, and for angles greater than this, the shape of the transmitted flexural wave was found to be very insensitive to the angle of intersection of the two bars.

Dynamic Tangent Moduli of Polyvinyl Chloride  
Subjected to Large Strains

Bernard J. Reilly, Jr.

Abstract

The data presented in this report describe the changes in the linear dynamic viscoelastic properties of polyvinyl chloride when it has been subjected to large one-dimensional strains. Dynamic tests were carried out on specimens which had undergone large quasistatic deformations in conditions of both creep and relaxation. At various times during the tests, small oscillatory deformations were superimposed on the large deformations. By measuring the resonant frequencies and decay curves, the values of the complex shear modulus  $G^*$  and complex Young's modulus  $E^*$  were determined.

The large axial deformations were produced by a machine made specifically for these tests. The small oscillatory deformations were produced by a coil-magnet system arranged so that either lateral or torsional vibrations could be set up. The detection and measurement of the vibrations was also carried out by a coil-magnet system. An oscilloscope was employed to display the detected signals. The resonant frequency could be found and a decay-curve picture could be recorded from the oscilloscope traces.

The shear modulus was determined by data from the torsional vibrations, while the Young's modulus was extracted from data with lateral vibrations.

Tech. Report No. 17

Elastic Waves Produced by Longitudinal  
Impact on a System with Symmetrically Branched Rods

by

James P. Lee  
Division of Applied Mathematics  
Brown University  
Providence, R.I.

Abstract

The problem of the propagation of a longitudinal elastic wave which is travelling along a thin uniform rod at the end of which a branched, symmetrically arranged system of rods is joined is treated analytically. The symmetry condition greatly simplifies the analysis and enables the problem to be treated as a two-dimensional one. The theory used for the propagation of longitudinal waves along the rods is the simple one-dimensional one while flexural wave propagation is treated by the Timoshenko theory. The numerical results obtained with this analysis are found to agree well with experimental observations.

Tech. Report. No. 18

# Experimental Demonstration of a "Viscoelastic Crossover"

Phenomenon. The Reflection of Stress Pulses at the Boundary Between Nearly-Elastic and Lossy Viscoelastic Media whose Impedances are Matched. By Mark B. Moffett

## Abstract

An experiment has been conducted which demonstrates a viscoelastic "crossover" effect originally described theoretically by Kolsky and Lee. They discussed the pulse shapes to be expected when an incident dc stress pulse of known shape is reflected from the interface between two viscoelastic solids, both of which are described by a constant -  $\tan \delta$  model, i.e., the loss angle of the complex modulus of each material is assumed to be independent of frequency. (This model is approximately valid for many polymers at temperatures removed from their glass-rubber transitions, i.e. either when they are in the glassy state or in the rubbery state.) If the loss angles for the two materials are different, there exists a frequency at which the magnitudes of their characteristic impedances,  $\sqrt{\rho|E|}$  ( $\rho$  being the density and  $E$  being the complex modulus), are equal. This is termed the crossover frequency. When the incident medium is less lossy than the reflecting medium, one expects the reflected pulses to:

- 1) have the same shape and sign if the fundamental pulse frequency  $1/2(t_2 - t_1)$  (where  $t_2 - t_1$  is the pulse duration) is much larger than the crossover frequency,
- 2) have the same shape but opposite sign if  $1/2(t_2 - t_1)$  is much smaller than the crossover frequency,
- or 3) undergo significant changes in pulse shape if

$1/2(t_2-t_1)$  is in the neighborhood of the crossover frequency.

In the experiment, dc stress pulses of length  $t_2-t_1 \approx 130 \mu\text{sec}$  were reflected from the interface between two rods of different viscoelastic materials. The incident medium was a low-loss material (polystyrene) and the reflecting medium was a lossy material (unplasticized polyvinyl chloride). By varying the temperature of the system from  $21^\circ\text{C}$  to  $55^\circ\text{C}$ , the reflected pulse shape changed its character in the expected manner from a positive pulse with a tilt to a negative pulse with a tilt which became flatter at the highest temperatures. Comparison of the experimental pulse shapes with those computed on the basis of a constant  $\tan \delta$  for the reflecting medium and zero  $\tan \delta$  for the incident medium suggested that the crossover frequency varied from about 300 Hz to about 100 kHz and that  $\tan \delta$  varied from about 0.05 to about 0.03 as the temperature changed from  $21^\circ\text{C}$  to  $55^\circ\text{C}$ . These results, while in rough agreement with measurements of the complex modulus of the PVC using a resonant technique, await confirmation by more refined measurements of the properties of both materials.

Some Recent Experimental Investigations in Stress-Wave  
Propagation and Fracture

Abstract

This paper describes preliminary results of three distinct experimental programs at present being carried out by the author and his colleagues at Brown University. These are:

- (a) Rubber is found to have a stress-strain curve which is concave upwards for tensile strains, i.e., the tangent modulus increases with increasing strain and also the attenuation becomes small at high strains. As a result of this, tensile pulses of reasonably large amplitudes are found to develop shock fronts when they are propagated along rubber filaments which are already subjected to large tensile pre-strains. Experiments illustrating this effect are described, and a discussion of the conditions for such shock-wave formation is given.
- (b) When a fracture occurs in a brittle solid, there is a sudden local change in stress and a compressive stress pulse is propagated away from the crack. Experiments are described concerning the generation of such pulses for Hertzian fractures, simple tensile fractures, and flexural fractures. The theoretically predicted pulse shapes are compared with those observed experimentally.
- (c) When a longitudinal stress pulse is reflected at normal incidence at the boundary between two solids of different properties, two pulses are, in general, produced. A reflected pulse is sent back, and a transmitted pulse is propagated in the

second medium. When one or both of the solids are viscoelastic, both the transmitted and reflected pulses have shapes which differ from that of the incident pulse. In particular, if the values of the acoustic impedances cross at some frequency  $\omega$ , called the cross-over frequency, dc pulses of duration comparable with  $1/\omega$  are reflected as s-shaped ones. Experiments which show this phenomenon are described.



Reflection of Pulses at the Interface Between an Elastic Rod  
and an Elastic Half-Space

Serge Boucher

Abstract

The problem of the reflection of elastic stress pulses at the interface between a rod and a half-space is treated theoretically and experimentally. The axis of the rod is taken to be perpendicular to the free surface of the half-space and its flat end to be in frictionless contact with this surface.

From the Fourier analysis of the incident and reflected pulses in the rod, the dynamic coefficient of reflection is computed. It is found that the accuracy with which the dynamic properties of the material of the half-space can be computed is limited by the precision with which the values of the Fourier components of the pulse are known.

An experimental investigation of stress waves in rods of a fiber-reinforced composite by Peter J. Arsenaux, Jr.

Abstract

The propagation of pulses in square bars of glass-epoxy fiber reinforced material was observed experimentally. The fiber orientation was varied from parallel to perpendicular to the bar's axis. Phase velocities were compared with those predicted from taking an effective Young's modulus for the bar. Geometric dispersion was considered by examining sinusoidal vibrations of composite rods of circular cross-section. Dispersion curves were predicted based on a lateral inertia correction and measured using a resonance method. The experimental results are not very extensive but nevertheless sufficient for qualitative conclusions to be advanced. Damping measurements were taken from oscilloscope records of free oscillations of the round composite rods. Again the results are not as complete as one would like, but some conclusions can be drawn.